NONLINEAR EXCHANGE OF MASS BETWEEN A GAS AND A LIQUID RUNOFF FILM. 2. ASYMPTOTIC ANALYSIS

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We propose an asymptotic method for the solution of the problem concerned with the determination of the rate of nonlinear mass exchange between a gas and a runoff fluid film, when the process is limited to the transport of mass in the gaseous phase. We have derived equations for the distributions of velocities and concentrations both in the liquid and the gas. The results from asymptotic theory are compared with the results of numerical analysis and against the experimental data.

It was demonstrated in the first part of [1] that in order to determine the distribution of velocities and concentrations in a gas and in a liquid in the case of nonlinear mass exchange between the gas and the runoff liquid film, limited to the transfer of mass in the gaseous phase, it is the following problem that has to be solved:

$$\begin{gathered}\Phi''' + \frac{1}{\varepsilon}\Phi\Phi'' = 0,\ \ \Psi'' + \varepsilon\Phi\Psi' = 0;\\ \Phi(0) = -\frac{\theta_3}{\varepsilon}\Psi'(0),\ \ \Phi'(0) = \frac{3\theta_1}{\varepsilon},\ \ \Phi'(\infty) = \frac{2}{\varepsilon},\\ \Psi(0) = 1,\ \ \Psi(\infty) = 0.\end{gathered} \tag{1}$$

Here the parameters θ_1 and θ_3 are small for those cases of practical interest. This enables us to find the solution for (1) by the method of perturbations, representing the unknown functions in the form of the following series:

$$A = A_0 + \theta_1 A_1 + \theta_3 A_3 + \theta_1^2 A_{11} + \theta_3^2 A_{33} + \theta_1\theta_3 A_{13} + \ldots,$$

where A is the vector function A = (Φ, Ψ).

Zeroth Approximations. These approximations are obtained from (1), provided that we assume that $\theta_1 = \theta_3 = 0$. The derived problems have known solutions [2, 3]:

$$\begin{gathered}\Phi_0(\eta) = f(z),\ \ \Psi_0(\eta) = 1 - \frac{1}{\varphi_0}\int_0^z E(\varepsilon, p)\,dp,\\ z = \frac{2}{\varepsilon}\eta,\end{gathered} \tag{2}$$

where φ_0 is the $\bar{Sc}$ [4, 5] function:

$$\varphi_0 = \int_0^\infty E(\varepsilon, p)\,dp \approx 3{,}01\,\bar{Sc}^{-0,35};$$

$$E(\varepsilon, p) = \exp\left[-\frac{\varepsilon^2}{2}\int^p f(s)\,ds\right],\ \ \varepsilon = \sqrt{\bar{Sc}},$$

while f is the solution of the problem

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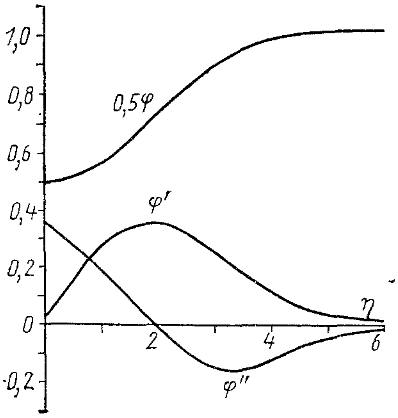

Fig. 1

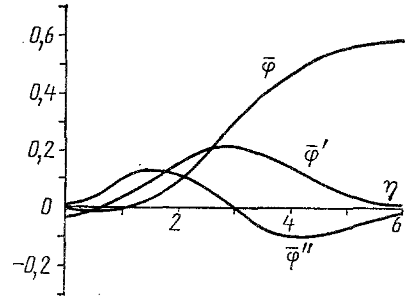

Fig. 2

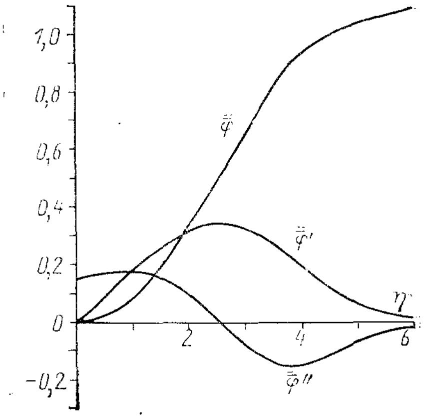

Fig. 3

Fig. 1. The function φ in (6) and its derivatives.

Fig. 2. The function $\bar{\varphi}$ in (9) and its derivatives.

Fig. 3. The function $\bar{\bar{\varphi}}$ in (15) and its derivatives.

$$
\begin{gathered}
2f''' + ff'' = 0; \\
f(0) = f'(0) = 0, \ f'(\infty) = 1,
\end{gathered} \tag{3}
$$

which has been tabulated in [6].

Effect of Kinematic Phase Interaction. Kinematic phase interaction expresses the continuity of velocities at the phase boundary and is taken into consideration by the parameter θ_1. If we substituted the asymptotic series (A) into (1) and if we equate terms proportional to θ_1, in first approximation we obtain the familiar problems [2, 3], whose solutions have the form

$$
\begin{gathered}
\Phi_1(\eta) = \frac{3}{2\alpha} f'(z), \ \alpha = f''(0) = 0{,}33205; \\
\Psi_1(\eta) = \frac{3}{2\alpha\varphi_0}\left[1 - E(\varepsilon, z) - \frac{1}{\varphi_0}\int\limits_0^z E(\varepsilon, p)\,dp\right].
\end{gathered} \tag{4}
$$

The second approximation is obtained if those terms proportional to θ_1^2 are equated. Thus, we obtain the known problems [2, 3] whose solutions have the form

$$
\Phi_{11}(\eta) = \frac{9}{4} F(z),
$$

$$\Psi_{11}(\eta) = \left(-\frac{9\varepsilon^2\varphi_2}{8\varphi_0^2} - \frac{9}{4\alpha^2\varphi_0^3} + \frac{9\varepsilon^4\varphi_1}{32\alpha^2\varphi_0^2}\right)\int_0^z E(\varepsilon,\ p)\,dp +$$
$$+\frac{9\varepsilon^2}{8\varphi_0}\int_0^z\left[\int_0^p F(s)\,ds\right]E(\varepsilon,\ p)\,dp + \frac{9}{4\alpha^2\varphi_0^2}\left[1 - E(\varepsilon,\ z)\right] - \tag{5}$$
$$-\frac{9\varepsilon^4}{32\alpha^2\varphi_0}\int_0^z f^2(p)\,E(\varepsilon,\ p)\,dp,$$

where φ_1 and φ_2 are $\tilde{\mathrm{Sc}}$ [4, 5] functions:

$$\varphi_1 = \int_0^\infty f^2(p)\,E(\varepsilon,\ p)\,dp \approx 3{,}011\,\tilde{\mathrm{Sc}}^{-1,608},$$
$$\varphi\quad \int_0^\infty\left[\int_0^p F(s)\,ds\right]E(\varepsilon,\ p)\,dp \approx 3{,}052\,\tilde{\mathrm{Sc}}^{-1,283},$$

while F is the solution of the problem

$$2F''' + fF'' + f''F = -\frac{1}{\alpha^2}f'f''';$$
$$F(0) = F'(0) = F'(\infty) = 0,$$

which has been tabulated in [6].

Effect of Nonlinearity. The nonlinear effect is associated with the concentration gradient, i.e., it depends significantly on the value of the parameter θ_3. When $\theta_3 < 10^{-1}$ we observe slight nonlinearity and its effect serves as the first approximation with respect to the parameter θ_3. If we equate the terms proportional to θ_3, we obtain the familiar problems [7] whose solutions have the form

$$\Phi_3(\eta) = \frac{2}{\varepsilon^2\varphi_0}\varphi(z),\quad \Psi_3(\eta) = \frac{1}{\varphi_0^2}\int_0^z\left[\int_0^p \varphi(s)\,ds\right]E(\varepsilon,\ p)\,dp - \frac{\varphi_3}{\varphi_0^3}\int_0^z E(\varepsilon,\ p)\,dp, \tag{6}$$

where φ_3 is the $\tilde{\mathrm{Sc}}$ function and is defined in Table 1:

$$\varphi_3 = \int_0^\infty\left[\int_0^p \varphi(s)\,ds\right]E(\varepsilon,\ p)\,dp \approx 6{,}56\,\tilde{\mathrm{Sc}}^{-0,8},$$

while φ is the solution to the problem

$$2\varphi''' + f\varphi'' + f''\varphi = 0;$$
$$\varphi(0) = 1,\ \varphi'(0) = \varphi'(\infty) = 0,$$

obtained by a numerical method (Fig. 1).

Pronounced nonlinearity is observed when $\theta_3 < 10^{-1}$ and is taken into consideration by the parameter θ_3^2. This effect is the second approximation with respect to the parameter θ_3 and is determined from (1) after having equated the terms proportional to θ_3^2:

$$\Phi_{33}''' + \frac{1}{\varepsilon}(\Phi_0\Phi_{33}'' + \Phi_{33}\Phi_0'' + \Phi_3\Phi_3'') = 0;$$
$$\Phi_{33}(0) = -\frac{1}{\varepsilon}\Psi_3'(0) = \frac{2\varphi_3}{\varepsilon^2\varphi_0^3},\quad \Phi_{33}'(0) = \Phi_{33}'(\infty) = 0. \tag{7}$$

After substitution of (2) and (6) into (7) we obtain

$$\Phi_{33}''' + \frac{1}{\varepsilon}f\Phi_{33}'' + \frac{4}{\varepsilon^3}f''\Phi_{33} = -\frac{16}{\varepsilon^7\varphi_0^2}\varphi\varphi''. \tag{8}$$

The solution for (8) has the form

TABLE 1. Values of the Functions φ_3, Obtained Through Integration and by Means of an Approximation Relationship

$\bar{Sc}$	φ_3	$6{,}56\ \bar{Sc}^{-0{,}8}$
0,1	32,53	41,39
0,2	22,93	23,76
0,5	11,58	11,42
0,8	7,86	7,84
1,0	6,56	6,56
1,2	5,66	5,67
1,5	4,74	4,74
1,8	4,11	4,10
2,0	3,79	3,77
5,0	1,90	1,81
10,0	1,15	1,04

TABLE 2. Values of the Functions φ_{33}, Obtained Through Integration and by Means of an Approximation Relationship

$\bar{Sc}$	φ_{33}	$24\bar{Sc}^{-1{,}3}$
0,1	263,80	478,86
0,2	163,97	194,48
0,5	59,37	59,09
0,8	32,12	32,08
1,0	23,99	24,00
1,2	18,94	18,93
1,5	14,24	14,17
1,8	11,31	11,18
2,0	9,91	9,75
5,0	3,28	2,96
10,0	1,48	1,20

TABLE 3. Values of the Functions φ_{33}, Obtained Through Integration and by Means of an Approximation Relationship

$\bar{Sc}$	$\bar{\varphi}_{33}$	$0{,}326\bar{Sc}^{-1{,}63}$
0,1	4,29	13,91
0,2	2,66	4,49
0,5	0,924	1,009
0,8	0,462	0,469
1,0	0,326	0,326
1,2	0,243	0,242
1,5	0,168	0,168
1,8	0,123	0,125
2,0	0,103	0,105
5,0	0,0186	0,0236
10,0	0,0040	0,0076

$$\Phi_{33}(\eta) = \frac{2\varphi_3}{\varepsilon^2\varphi_0^3}\,\varphi(z) - \frac{4}{\varepsilon^4\varphi_0^2}\,\bar{\varphi}(z), \tag{9}$$

where $\bar{\varphi}$ is the solution of the problem

$$2\bar{\varphi}''' + f\bar{\varphi}'' + f''\bar{\varphi} = \varphi\varphi'',$$
$$\bar{\varphi}(0) = \bar{\varphi}'(0) = \bar{\varphi}'(\infty) = 0,$$

obtained numerically (Fig. 2).

Let us write out the distribution of concentrations:

$$\Psi_{33}'' + \varepsilon(\Phi_0\Psi_{33}' + \Phi_{33}\Psi_0' + \Phi_3\Psi_3') = 0;$$
$$\Psi_{33}(0) = \Psi_{33}(\infty) = 0. \tag{10}$$

Taking into consideration (2), (6), and (9), from (10) we find that

TABLE 4. Values of the Function φ_{13}, Obtained Through Integration and by Means of an Approximation Relationship

$\tilde{Sc}$	φ_{13}	$\tilde{Sc}^{-1,3}$
0,1	9,62	19,95
0,2	6,19	8,10
0,5	2,41	2,46
0,8	1,34	1,34
1,0	1,00	1,00
1,2	0,794	0,789
1,5	0,593	0,590
1,8	0,466	0,466
2,0	0,405	0,406
5,0	0,119	0,123
10,0	0,047	0,050

TABLE 5. Values of the Functions $\overline{\varphi}_{13}$, Obtained Through Integration and by Means of an Approximation Relationship

$\tilde{Sc}$	$\overline{\varphi}_{13}$	$4,18\tilde{Sc}^{-0,46}$
0,1	10,58	12,07
0,2	8,54	8,76
0,5	5,77	5,75
0,8	4,63	4,63
1,0	4,18	4,18
1,2	3,84	3,84
1,5	3,47	3,47
1,8	3,19	3,19
2,0	3,05	3,04
5,0	2,05	1,99
10,0	1,54	1,45

$$\Psi_{33}'' + \varepsilon f(z)\Psi_{33}' = \left[\frac{8\varphi_3}{\varepsilon^2\varphi_0^4}\varphi(z) - \frac{8}{\varepsilon^4\varphi_0^3}\overline{\varphi}(z) - \frac{4}{\varepsilon^2\varphi_0^3}\varphi(z)\int_0^z\varphi(s)\,ds\right]E(\varepsilon,\ z). \tag{11}$$

The solution to (11) has the form

$$\Psi_{33}(\eta) = \left(-\frac{2\varphi_3^2}{\varphi_0^5} + \frac{\varphi_{33}}{2\varphi_0^4} + \frac{2\overline{\varphi}_{33}}{\varepsilon^2\varphi_0^4}\right)\int_0^z E(\varepsilon,\ p)\,dp +$$

$$+\frac{2\varphi_3}{\varphi_0^4}\int_0^z\left[\int_0^p\varphi(s)\,ds\right]E(\varepsilon,\ p)\,dp - \frac{1}{2\varphi_0^3}\int_0^z\left[\int_0^p\varphi(s)\,ds\right]^2E(\varepsilon,\ p)\,dp - \frac{2}{\varepsilon^2\varphi_0^3}\int_0^z\left[\int_0^p\overline{\varphi}(s)\,ds\right]E(\varepsilon,\ p)\,dp, \tag{12}$$

where φ_{33} and $\overline{\varphi}_{33}$ are the $\tilde{Sc}$ functions:

$$\varphi_{33} = \int_0^\infty\left[\int_0^p\varphi(s)\,ds\right]^2 E(\varepsilon,\ p)\,dp \approx 24\,\tilde{Sc}^{-1,3},$$

$$\overline{\varphi}_{33} = \int_0^\infty\left[\int_0^p\overline{\varphi}(s)\,ds\right] E(\varepsilon,\ p)\,dp \approx 0{,}326\,\tilde{Sc}^{-1,63}$$

and are defined in Tables 2 and 3.

Strong Interaction Effect. For the cases in which $\theta_1 > 10^{-1}$ and $\theta_3 > 10^{-1}$ it is essential that we take into consideration in (1) the terms proportional to $\theta_1\theta_3$:

$$\Phi_{13}''' + \frac{1}{\varepsilon}(\Phi_0\Phi_{13}'' + \Phi_{13}\Phi_0'' + \Phi_1\Phi_3'' + \Phi_3\Phi_1'') = 0,$$

$$\Phi_{13}(0) = -\frac{1}{\varepsilon}\Psi_1'(0),\ \ \Phi_{13}'(0) = \Phi_{13}'(\infty) = 0. \tag{13}$$

Having substituted (2), (4), and (6) into (13), we obtain

$$\Phi_{13}''' + \frac{1}{\varepsilon}f\Phi_{13}'' + \frac{4}{\varepsilon^3}f''\Phi_{13} = -\frac{12}{\alpha\varepsilon^5\varphi_0}(f'\varphi'' + f'''\varphi);$$

$$\Phi_{13}(0) = \frac{3}{\alpha\varepsilon^2\varphi_0^2},\ \ \Phi_{13}'(0) = \Phi_{13}'(\infty) = 0. \tag{14}$$

The solution of (14) has the form

$$\Phi_{13}(\eta) = \frac{3}{\alpha\varepsilon^2\varphi_0^2}\varphi(z) - \frac{3}{\alpha\varepsilon^2\varphi_0}\overline{\overline{\varphi}}(z), \tag{15}$$

where $\overline{\overline{\varphi}}$ is the solution of the problem $2\overline{\overline{\varphi}}''' + f\overline{\overline{\varphi}}'' + f''\overline{\overline{\varphi}} = f'\varphi'' + f'''\varphi$, $\overline{\overline{\varphi}}(0) = \overline{\overline{\varphi}}'(0) = \overline{\overline{\varphi}}'(\infty) = 0$, numerically (Fig. 3).

Let us find the distribution of the concentrations

$$\Psi_{13}'' + \varepsilon(\Phi_0\Psi_{13}' + \Phi_{13}\Psi_0' + \Phi_1\Psi_3' + \Phi_3\Psi_1') = 0;\quad \Psi_{13}(0) = \Psi_{13}(\infty) = 0. \tag{16}$$

Bearing in mind (2), (4), (6), and (15), from (16) we obtain

$$
\Psi_{13}'' + \varepsilon f(z)\Psi_{13}' = \Big[\frac{12}{\alpha\varepsilon^2\varphi_0^3}\varphi(z) - \frac{6}{\alpha\varepsilon^2\varphi_0^2}\overline{\overline{\varphi}}(z) -
$$

$$
- \frac{3}{\alpha\varphi_0^2} f'(z)\int_0^z \varphi(s)\,ds + \frac{3\varphi_3}{\alpha\varphi_0^3} f'(z) - \frac{3}{\alpha\varphi_0^2} f(z)\varphi(z)\Big] E(\varepsilon,\ z); \tag{17}
$$

$$
\Psi_{13}(0) = \Psi_{13}(\infty) = 0.
$$

The solution of (17) has the form

$$
\Psi_{13}(\eta) = \Big(-\frac{9\varphi_3}{\quad} + \frac{3\varphi_{13}}{2\alpha\varphi_0^3} + \frac{3\overline{\varphi}_{13}}{2\alpha\varphi_0^3}\Big)\int_0^z E(\varepsilon,\ p)\,dp +
$$

$$
+ \frac{3}{\alpha\varphi_0^3}\int_0^z\Big[\int_0^p \varphi(s)\quad s\Big] E(\varepsilon,\ p)\,dp - \frac{3}{2\alpha\varphi_0^2}\int_0^z\Big[\int_0^p \varphi(s)\,ds\Big] E(\varepsilon,\ p)\,dp + \tag{18}
$$

$$
+ \frac{3}{2\alpha\varphi_0^2} E(\varepsilon,\ .)\int_0^z \varphi(s)\,ds - \frac{3}{2\alpha\varphi_0^2}\int_0^z \varphi(p) E(\varepsilon,\ p)\,dp + \frac{3\varphi_3}{2\alpha\varphi_0^3}[1 - E(\varepsilon,\ z)],
$$

where φ_{13} and $\overline{\varphi}_{13}$ are the $\tilde{Sc}$ functions:

$$
\varphi_{13} = \int_0^\infty\Big[\int_0^p \overline{\overline{\varphi}}(s)\,ds\Big] E(\varepsilon,\ p)\,dp \approx \tilde{Sc}^{-1,3},
$$

$$
\overline{\varphi}_{13} = \int_0^\infty \varphi(p) E(\varepsilon,\ p)\,dp \approx 4{,}18\,\tilde{Sc}^{-0,46}
$$

and are defined in Tables 4 and 5.

Effect of Nonlinear Mass Transfer in the Gas on the Hydrodynamics of the Film. The nonlinear effects in the gas affect film hydrodynamics, since it was demonstrated in [1] that the distribution of velocities and thicknesses for the film depends on the distribution of the velocity and concentration in the gas.

The hydrodynamics of the film depends on the small parameter θ_2 which is found in [1] from the condition of continuity for the stress tensor at the phase boundary and takes into consideration the dynamic interaction of the liquid and the gas. Maintaining the level of accuracy for the approximations derived so far, we obtain the distribution for the velocities within the liquid in first approximation of the small parameter θ_2:

$$
U = -\frac{3}{2}Y^2 + \Big[3H + \theta_2\frac{\varepsilon}{4\sqrt{X}}\Phi_0''(0)\Big]Y,
$$

$$
V = -\frac{1}{2}\Big[3H' - \theta_2\frac{\varepsilon}{8X\sqrt{X}}\Phi_0''(0)\Big]Y^2, \tag{19}
$$

where H is found from

$$
H^3 = 1 - \theta_2\frac{\varepsilon}{8\sqrt{X}}\Phi_0''(0)H^2 + \theta_0\theta_3[\Psi_0'(0) + \theta_1\Psi_1'(0) + \theta_3\Psi_3'(0)](\sqrt{L_\infty} - \sqrt{X}). \tag{20}
$$

For purposes of determining H it is convenient to use the expansion

$$
H = 1 + \theta_2 H_2 + \theta_3 H_3 + \theta_3^2 H_{33} + \theta_1\theta_3 H_{13} + \ldots \tag{21}
$$

If we substitute (21) into (20) and equate terms with identical powers of θ_2, θ_3, θ_3^2 and $\theta_1\theta_3$, bearing in mind (2), (4), and (6), we find

$$
H_2 = -\frac{\alpha}{6\varepsilon\sqrt{X}},\quad H_3 = -\frac{2\theta_0}{3\varepsilon\varphi_0}(\sqrt{L_\infty} - \sqrt{X}),
$$

$$
H_{33} = -\frac{4\theta_0^2}{9\varepsilon^2\varphi_0^2}(\sqrt{L_\infty} - \sqrt{X})^2 - \frac{2\theta_0\varphi_3}{3\varepsilon\varphi_0^3}(\sqrt{L_\infty} - \sqrt{X}), \tag{22}
$$

$$
H_{13} = -\frac{\theta_0}{\alpha\varepsilon\varphi_0^2}(\sqrt{L_\infty} - \sqrt{X}).
$$

Substituting (2) and (22) into (19) and (21), in final form we obtain

$$U = -\frac{3}{2}Y^2 + \left(3H + \frac{\alpha\theta_2}{\varepsilon\sqrt{X}}\right)Y,$$

$$V = -\frac{1}{2}\left(3H' - \frac{\alpha\theta_2}{2\varepsilon X\sqrt{X}}\right)Y^2$$

and we determine H:

$$H = 1 - \frac{\alpha\theta_2}{6\varepsilon\sqrt{X}} - \frac{2\theta_0\theta_3}{2\varepsilon\varphi_0}(\sqrt{L_\infty} - \sqrt{X}) -$$

$$-\theta_3^2\left[\frac{4\theta_0^2}{9\varepsilon^2\varphi_0^2}(\sqrt{L_\infty} - \sqrt{X})^2 + \frac{2\theta_0\varphi_3}{3\varepsilon\varphi_0^3}(\sqrt{L_\infty} - \sqrt{X})\right] -$$

$$-\theta_1\theta_3\frac{\theta_0}{\alpha\varepsilon\varphi_0^2}(\sqrt{L_\infty} - \sqrt{X}).$$

Distribution of Velocities in the Gas. The above-derived results enable us to write expressions for the distribution of the velocities in the gas, as follows:

$$\tilde{U}(X, \tilde{Y}) = \frac{\varepsilon}{2}\Phi'(\eta) = f'(z) + \frac{3\theta_1}{2\alpha}f''(z) +$$

$$+\frac{9\theta_1^2}{4}F'(z) - \frac{2\theta_3}{\varepsilon^2\varphi_0}\varphi'(z) + \frac{2\theta_3^2\varphi_3}{\varepsilon^2\varphi_0^3}\varphi'(z) - \frac{4\theta_3^2}{\varepsilon^4\varphi_0^2}\bar{\varphi}'(z) + \frac{3\theta_1\theta_3}{2\alpha\varepsilon\varphi_0^2}\varphi'(z) - \frac{3\theta_1\theta_3}{2\alpha\varepsilon\varphi_0}\bar{\bar{\varphi}}'(z),$$

$$V(X, \tilde{Y}) = \frac{\varepsilon}{2\sqrt{X}}(\eta\Phi' - \Phi) = \frac{\varepsilon}{2\sqrt{X}}\left\{\left[\frac{2}{\varepsilon}\eta f'(z) - f(z)\right] +\right.$$

$$+\frac{3\theta_1}{2\alpha}\left[\frac{2}{\varepsilon}\eta f''(z) - f'(z)\right] + \frac{9\theta_1^2}{4}\left[\frac{2}{\varepsilon}\eta F'(z) - F(z)\right] + \tag{23}$$

$$+\frac{2\theta_3}{\varepsilon^2\varphi_0}\left[\frac{2}{\varepsilon}\eta\varphi'(z) - \varphi(z)\right] + \frac{2\theta_3^2\varphi_3}{\varepsilon^2\varphi_0^3}\left[\frac{2}{\varepsilon}\eta\varphi'(z) - \varphi(z)\right] -$$

$$\left.-\frac{4\theta_3^2}{\varepsilon^4\varphi_0^2}\left[\frac{2}{\varepsilon}\eta\bar{\varphi}'(z) - \bar{\varphi}(z)\right] + \frac{3\theta_1\theta_3}{\alpha\varepsilon^2\varphi_0^2}\left[\frac{2}{\varepsilon}\eta\varphi'(z) - \varphi(z)\right] - \frac{3}{\alpha\varepsilon^2\varphi_0}\left[\frac{2}{\varepsilon}\bar{\bar{\varphi}}'(z) - \bar{\bar{\varphi}}(z)\right]\right\}.$$

Effect of Mass-Exchange Direction on the Rate of Mass Transfer. The rate of mass transfer in the case under consideration is determined by the Sherwood number from [1]. In the new variables this expression assumes the form

$$\tilde{\mathrm{Sh}} = \frac{\tilde{\rho}^*}{\tilde{\rho}_0^*}\sqrt{\tilde{\mathrm{Pe}}}\,\Psi'(0), \tag{24}$$

where $\Psi'(0)$ can be obtained from (2), (4)-(6), (12), and (18):

$$\Psi'(0) = -\left[\frac{2}{\varepsilon\varphi_0} + \frac{3\theta_1}{\varepsilon\alpha\varphi_0^2} + \frac{2\theta_3\varphi_3}{\varepsilon\varphi_0^3} + \theta_1^2\left(\frac{9\varepsilon\varphi_2}{4\varphi_0^2} + \frac{9}{2\varepsilon\alpha^2\varphi_0^3} -\right.\right.$$

$$\left.\left.-\frac{9\varepsilon^3\varphi_1}{16\alpha^2\varphi_0^2}\right) + \theta_3^2\left(\frac{4\varphi_3^2}{\varepsilon\varphi_0^5} - \frac{\varphi_{33}}{\varepsilon\varphi_0^4} - \frac{4\bar{\varphi}_{33}}{\varepsilon^3\varphi_0^4}\right) + \theta_1\theta_3\left(\frac{9\varphi_3}{\alpha\varepsilon\varphi_0^4} - \frac{3\varphi_{13}}{\alpha\varepsilon\varphi_0^3} - \frac{3\bar{\varphi}_{13}}{\alpha\varepsilon\varphi_0^3}\right)\right]. \tag{25}$$

It is directly evident from (25) that the nonlinear effects increase the rate of absorption ($\theta_3 > 0$) and reduce the rate of desorption ($\theta_3 < 0$) relative to the rate that is obtained from the linear theory of mass transport ($\theta_3 = 0$). If we express the Sherwood number and the mass-transfer coefficients in terms of $\tilde{\mathrm{Sh}}_+$ and $\tilde{k}_+$ in the case of absorption ($\theta_3 > 0$), in terms of $\tilde{S}_-$ and $\tilde{k}_-$ in the case of desorption ($\theta_3 < 0$), and in terms of Sh_0 and $\tilde{k}_0$ in the cases in which $\theta_3 = 0$ (i.e., with small concentration gradients, when the rates of absorption and desorption are virtually equal), then from (24) and (25) we can obtain

TABLE 6. Velocity of the Secondary Flow and the Rate of Mass Transport, Obtained by a Numerical Method and on the Basis of Expressions (23) and (25)

θ_3	θ_1	$\Phi(0)$	$\Phi_a(0)$	$-\Psi'(0)$	$-\Psi_a'(0)$
0	0,0720	0	0	0,730	0,738
0,1	0,0723	0,0785	0,0784	0,785	0,794
—0,1	0,0718	—0,0682	—0,0687	0,682	0,690
0,2	0,0725	0,170	0,166	0,851	0,857
—0,2	0,0716	—0,128	—0,128	0,641	0,650
0,3	0,0730	0,280	0,264	0,932	0,929
—0,3	0,0714	—0,182	—0,177	0,605	0,618

$$\frac{\tilde{\mathrm{Sh}}_+ - \tilde{\mathrm{Sh}}_0}{\tilde{\mathrm{Sh}}_+ - \tilde{\mathrm{Sh}}_-} = \frac{\tilde{k}_+ - \tilde{k}_0}{\tilde{k}_+ - \tilde{k}_-} = \frac{1}{2} + \theta_3 \frac{\frac{1}{\varepsilon^2\varphi_0^2}\left(4\varepsilon^2\varphi_3^2 - \varepsilon^2\varphi_0\varphi_{33} - 4\varphi_0\bar{\varphi}_{33}\right)}{4\varphi_3 + \frac{6\theta_1}{\alpha\varphi_0}\left(3\varphi_3 - \varphi_0\varphi_{13} - \varphi_0\bar{\varphi}_{13}\right)}. \tag{26}$$

Expression (26), which is a direct consequence of the nonlinear theory of mass transfer, can be verified directly on the basis of experimental data for the coefficient of mass transfer in the absorption and desorption of a readily soluble gas in the presence of large and small concentration gradients.

Comparison of the Asymptotic-Theory Results with the Results from Numerical Analysis and Against Experimental Data. Table 6 shows the results for the secondary-flow velocity $\Phi(0)$ and the mass-transfer rate $\Psi'(0)$ (obtained numerically) and the results $\Phi_a(0)$ and $\Psi_a'(0)$ from asymptotic theory, obtained through expressions (23) and (25). We can see from the table that the accuracy of approximation for asymptotic theory is adequate for practical calculations of the kinetics of nonlinear mass transfer.

The difference in the rate of mass transfer in the processes of absorption and desorption for readily soluble gases was observed experimentally by a number of authors [8-11]. In all of these cases this was explained by the Marangoni effect, i.e., the hydrodynamic effect resulting from initiation of a secondary flow whose velocity is tangential to the interphase surface. This effect is generated by the gradient of surface tension that is due to nonuniform distribution of temperatures and (or) concentrations at the interphase surface. In the present study we propose a theory which can explain these experimental results through the nonlinear effects resulting from the induced secondary flows whose velocity is normal to the phase boundary. Clarification of the mass-transfer mechanism in cases of intense mass exchange obviously calls for experimental data on absorption and desorption and the comparative analysis of these data by means of the theory of nonlinear mass transfer and the Marangoni effect. In this regard it is interesting also to determine the influence exerted by the effect of normal secondary flows (resulting from intense mass exchange) on the hydrodynamic stability of liquid and gas flows at the interphase surface.

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